9.5 Graph and Write Equations of Hyperbolas

Goal • Graph and write equations of hyperbolas.

Your Notes

VOCABULARY

Hyperbola The set of all points P such that the difference of the distances between P and two fixed points, called the foci, is a constant

Foci Two fixed points in a hyperbola

Vertices The points of intersection of a hyperbola and the line through the foci

Transverse Axis The line segment that connects the vertices of a hyperbola

Center The midpoint of the transverse axis

STANDARD EQUATION OF A HYPERBOLA WITH CENTER AT THE ORIGIN

Equation	Transverse Axis	Asymptotes	Vertices
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	Horizontal	$y = \pm \frac{b}{a} x$	$(\pm a, 0)$
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Vertical	$y = \pm \frac{a}{b} x$	$(0, \pm a)$

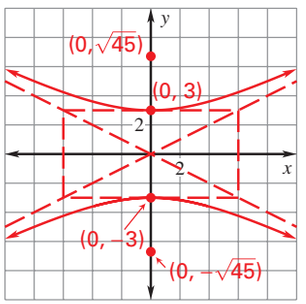
The foci lie on the transverse axis, c units from the center, where $c^2 = a^2 + b^2$.

Your Notes

Example 1 ***Graph an equation of a hyperbola***

Graph $36y^2 - 9x^2 = 324$. Identify the vertices, foci, and asymptotes of the hyperbola.

Solution



1. **Rewrite** the equation in standard form.

   $\frac{y^2}{9} - \frac{x^2}{36} = 1$

2. **Identify** the vertices, foci, and asymptotes. Note that $a^2 =$ 9 and $b^2 =$ 36, so $a =$ 3 and $b =$ 6. The y^2-term is positive, so the transverse axis is vertical and the vertices are $(0, \pm 3)$. Find the foci.

   $c^2 = a^2 + b^2 =$ $3^2 + 6^2$ $=$ 45, so $c = \sqrt{45}$

   The foci are at $(0, \pm\sqrt{45}) \approx (0, \pm 6.7)$.

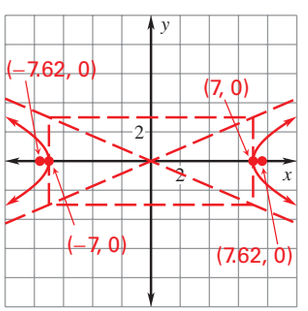
   The asymptotes are $y = \pm\frac{a}{b}x$, or $y =$ $\pm\frac{1}{2}x$.

3. **Draw** the hyperbola. Draw a rectangle centered at the origin that is $2a =$ 6 units high and $2b =$ 12 units wide. The asymptotes pass through opposite corners of the rectangle. Then, draw the hyperbola passing through the vertices and approaching the asymptotes.

✔ ***Checkpoint*** **Graph the equation. Identify the vertices, foci, and asymptotes of the hyperbola.**

1. $\frac{x^2}{49} - \frac{y^2}{9} = 1$

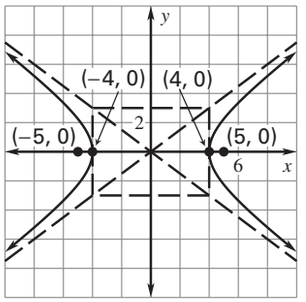
   vertices: $(\pm 7, 0)$
   foci: $(\pm 7.62, 0)$
   asymptotes: $y = \pm\frac{3}{7}x$



Your Notes

Example 2 *Write an equation of a hyperbola*

Write an equation of the hyperbola with foci at (−5, 0) and (5, 0) and vertices at (−4, 0) and (4, 0).



The foci and vertices lie on the x-axis equidistant from the origin, so the transverse axis is horizontal and the center is the origin. The foci are each 5 units from the center, so $c =$ 5. The vertices are each 4 units from the center, so $a =$ 4.

Because $c^2 = a^2 + b^2$, you have $b^2 = c^2 - a^2$. Find b^2.

$b^2 = c^2 - a^2 =$ $5^2 - 4^2$ $=$ 9

Because the transverse axis is horizontal, the standard form of the equation is as follows:

$\frac{x^2}{4^2} - \frac{y^2}{9} = 1$ **Substitute 4 for a and 9 for b^2.**

$\frac{x^2}{16} - \frac{y^2}{9} = 1$ **Simplify.**

✔ ***Checkpoint*** **Write an equation of the hyperbola with the given foci and vertices.**

2. Foci: (0, −8), (0, 8)
Vertices: (0, −5), (0, 5)

$\frac{y^2}{25} - \frac{x^2}{39} = 1$

Your Notes

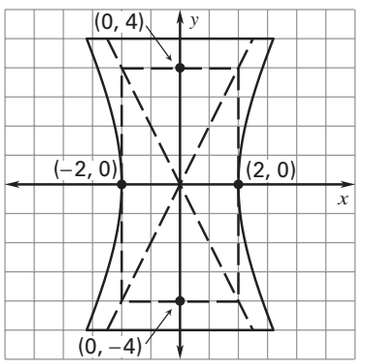
Example 3 ***Solve a multi-step problem***

Lamp The diagram shows the hyperbolic cross section of a lamp. Write an equation for the cross section of the lamp. The lamp is 10 inches high. How wide is the base?

Solution

1. From the diagram, $a =$ 2 and $b =$ 4.

Because the transverse axis is horizontal, an equation for the cross section of the lamp is $\frac{x^2}{2^2} - \frac{y^2}{4^2} = 1$, or $\frac{x^2}{4} - \frac{y^2}{16} = 1$.



2. Find the x-coordinate at the lamp's bottom edge. Because the lamp is 10 inches tall, substitute $y =$ 5 into the equation and solve.

$$\frac{x^2}{4} - \frac{5^2}{16} = 1$$

$$x^2 = 10.25$$

$$x \approx 3.20$$

So, the lamp has a width of $2x$ or $2($ 3.20 $) =$ 6.40 inches.

Checkpoint **Complete the following exercise.**

3. Write an equation for the hyperbolic cross section of the lamp in Example 3 if the vertices are at $(\pm 3, 0)$ and the foci are at $(\pm 5, 0)$. If the lamp is 15 inches high, how wide is the base?

$\frac{x^2}{9} - \frac{y^2}{16} = 1$

12.75 in.

Homework